
SIMULTANEOUS HEAT AND MASS TRANSFER DURING EVAPORATION FROM A FILM OF LIQUID INTO THE TURBULENT GAS

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The paper presents results obtained by processing a series of published experimental data on heat and mass transfer during evaporation of pure liquids from the freeboard of a liquid film into the turbulent gas phase. The data has been processed on the basis of the earlier theory of mechanism of heat and mass transfer. In spite of the fact that this process exhibits a strong Stefan's flow, the results indicate that with a proper definition of the driving forces the agreement between theory and experiment is very good.

The subject of investigation of this study has been the simultaneous transport of mass and heat occurring on direct contact of the freeboard of liquid with the turbulent gas. This process has been frequently utilized in industrial practice for saturation of gases and cooling of gases or liquids. Under certain circumstances also drying falls into this category. The available papers¹⁻⁵ dealing with evaporation are all of experimental character and concern mainly evaporation from the film of liquid flowing down an internal surface of a circular tube into the gas flowing inside the tube. The column proper, where both phases were contacted, was a vertical tube 20 to 29 mm in diameter of the effective length 800 to 1 200 mm. The upper end of the tube was equipped with an overflow chamber for the feed of liquid forced onto the inner wall of the tube. The lower end of the tube was provided with a slot for draining liquid. The tube was mounted between a steady sections of equal-diameter tubes with the aim to create and maintain a desired hydrodynamic regime in the flowing gas. All authors used air as the flowing gas and water as the evaporating liquid. Gilliland and Sherwood¹ evaporated also a number of other liquids of various physical properties. Some authors^{1,5} worked not only under the counter-current arrangement of the flows but also with the concurrent flows and under various pressures¹. Cairns and Roper³ and Yoshida and Hyodo⁴ worked with an increased content of vapours in the entering gas up to the superheated steam in the absence of air. In the processing of experimental data the authors assumed the existence of the Stefan flow⁶ and correlated their data by relationships of the Nusselt or Chilton-Colburn type.

THEORETICAL

The aim of this work has been to verify on the basis of experimental data the utility of the earlier derived relationship⁷ for the calculation of the mass and heat transfer coefficient for evaporation when both processes take place simultaneously and when the Stefan flow plays an important role.

Prior to the processing of data proper it is necessary to give precision to certain questions regarding the hydrodynamics of the flow in the system and the definitions of the driving forces for both phenomena.

Gilliland and Sherwood¹ found out, after processing their experimental data for the co- and counter-current flow of fluids, that both sets of data can be correlated provided the Reynolds criterion for the flow of gas is based on the velocity relative to the wall of the column and not to the trickling liquid film as may have been expected. The authors thought that owing to the used effective length of the tube (1 170 mm) the hydrodynamic conditions in the bulk phase did not correspond to the change of conditions on the wall. On the contrary, Hikita and coworkers⁵ found that an adequate correlation of data for the co- and counter-current arrangement of the flow can be achieved by respecting the relative motion of the gas phase with respect to the liquid surface of the liquid film. We have processed data both ways and the analysis have shown that for the large majority of results the approach respecting the relative motion of the gas phase and the liquid film provides a better fit. For this reason we shall confine ourselves in the following to this latter approach.

For the calculation of the thickness of the liquid film we used Nusselt's relation⁸

$$h = [3v_1\Gamma/(g \cdot \rho_1)]^{1/3} \quad (1)$$

valid for $Re_1 \leq 1\ 600$ which is accord with the given conditions.

The surface velocity of the liquid film was computed from Mjasnikov's expression⁹

$$v_1 = 0.485(gv_1)^{0.33} Re_1^{0.6} \quad (2)$$

valid in the region $280 \leq Re_1 \leq 1\ 600$ which is also in accord with the given experimental conditions.

The Reynolds number for the gas phase, defined with respect to the surface of the liquid film, has the following form

$$Re_{gr} = v_{gr}(d - 2h)/v_g \quad (3)$$

where

$$v_{gr} = v_g \pm v_1 \quad (4)$$

In Eq. (4) the plus sign pertains to the counter-current and the minus sign to the co-current arrangement of the flows.

From Eqs (3) and (4) it follows

$$Re_{gr} = [(v_g d / v_g)(1 - 2h/d)] \pm v_l(1 - 2h/d) d / v_g. \quad (5)$$

As to the area of the interfacial surface between the gas phase and the liquid film we shall assume, in accord with Brauer¹⁰, that the latter is negligibly affected by waviness of the liquid film.

In view of the fact that evaporation takes place under the marked effect of the Stefan flow, we shall use for the calculation of the mass transfer coefficient in the gas phase the appropriate relation taking the following form

$$dN = k_g(P/p_{a1n})(c^* - c_b) dA \quad (6)$$

or for the whole surface, after taking log-mean driving force

$$N = k_g \left\{ \left[\frac{P}{p_{a1n}} (c^* - c_b) \right]_1 - \left[\frac{P}{p_{a1n}} (c^* - c_b) \right]_2 \right\} \cdot A / \ln \left\{ \left[\frac{P}{p_{a1n}} (c^* - c_b) \right]_1 / \left[\frac{P}{p_{a1n}} (c^* - c_b) \right]_2 \right\} \quad (7)$$

where

$$p_{a1n} = (p_{ab} - p_{ai}) / \ln(p_{ab}/p_{ai}) \quad (8)$$

and

$$c^* = p_v^* / (RT_f) \quad (a), \quad c_b = p_{vb} / (RT_f). \quad (b) \quad (9a,b)$$

With the definition of the driving force in Eq. (7) the definition and the physical concept of the mass transfer coefficient remains the same as in those cases where the Stefan's flow is negligible. Thus the mass transfer coefficients should obey the earlier derived theoretical relationship⁷.

The majority of data used for the calculation of the heat transfer coefficient were obtained under the conditions when the temperature of liquid did not change after passing the column, or changed only a little. Under the assumption that the temperature of liquid remains constant and under the assumption of negligible radiation we may write the thermal balance in the form

$$(G_g c_{pg} + G_v c_{pv}) d(\Delta t) + (r + c_{pv} \Delta t) dG_v = 0 \quad (10)$$

where

$$\Delta t = t_g - t_l \quad (11)$$

and

$$dG_v = (\alpha/r) \Delta t \, dA. \quad (12)$$

The coefficient of the differential in the first term on the left hand side of Eq. (10) may be written in the form

$$G_g c_{pg} + G_v c_{pv} = G_g c_{pg} + G_{v1} c_{pv} + (G_v - G_{v1}) c_{pv}. \quad (13)$$

From the overall balance there follows

$$G_g c_{pg} + G_{v1} c_{pv} = (G_{v2} - G_{v1}) (r + c_{pv} \Delta t_2) / (\Delta t_1 - \Delta t_2). \quad (14)$$

Integration of Eq. (10) in the limits between Δt_1 and Δt or between G_{v1} and G_v yields after some rearrangement

$$G_v - G_{v1} = (G_g c_{pg} + G_{v1} c_{pv}) (\Delta t_1 - \Delta t) / (r + c_{pv} \Delta t). \quad (15)$$

Using Eqs (11) to (15), Eq. (10) may be arranged to the form

$$\left\{ (G_{v2} - G_{v1}) (1 + c_{pv} \Delta t_1 / r) (1 + c_{pv} \Delta t_2 / r) r^2 / (\Delta t_1 - \Delta t_2) \right\} d(\Delta t) / [(r + c_{pv} \Delta t)^2] \Delta t = -(\alpha/r) \, dA \quad (16)$$

or after integration to

$$(G_{v2} - G_{v1}) r = \alpha \cdot \Delta t_{in} A, \quad (17)$$

where

$$\Delta t_{in} = (\Delta t_1 - \Delta t_2) / \left\{ (1 + c_{pv} \Delta t_1 / r) (1 + c_{pv} \Delta t_2 / r) \cdot \ln \left[(\Delta t_1 / \Delta t_2) (1 + c_{pv} \Delta t_2 / r) / (1 + c_{pv} \Delta t_1 / r) \right] - c_{pv} (\Delta t_1 - \Delta t_2) / r \right\}. \quad (18)$$

The left hand side of Eq. (17) represents the latent heat necessary to evaporate the transferred mass and hence the heat that under the experimental conditions would be transported from the gas across the interface by convection. Eq. (17) defines thus the appropriate driving force for the process in the form given in Eq. (18).

In case that radiation plays a significant role and the temperature of liquid varies, one can write with sufficient accuracy Eq. (17) in the form

$$Q_c = (G_{v2} - G_{v1}) r - Q_R - Q_1 = \alpha \Delta t_{in} A \quad (19)$$

where Δt_{in} is defined by Eq. (18).

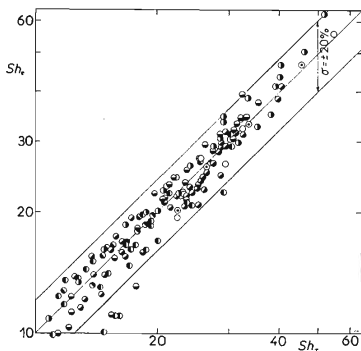


FIG. 1

Comparison of experimental and theoretical values of the Sherwood number: Gilliland and Sherwood¹ counter-current flow: water — ●, toluene — ○, ethyl acetate — ⊙, isopropanol — ⊕; Barnett and Kobe² counter-current flow: water — ●; Cairns and Roper³ counter-current flow: water — ●; Yoshida and Hyodo⁴ counter-current flow: water — ●; Hikita and coworkers⁵ counter-current flow: water — ●; Hikita and coworkers⁵ co-current flow: water — ●

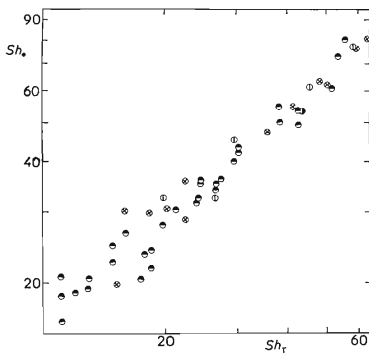


FIG. 2

Comparison of experimental and theoretical values of the Sherwood number for the data of Gilliland and Sherwood and co-current flow: water — ●, toluene — ○, ethyl acetate — ⊙, isopropanol — ⊕

RESULTS

The calculation of the experimental value of the mass transfer coefficient, or the Sherwood and the Nusselt number was always based on the rate of evaporation of liquid necessitating a certain intensity of both examined processes. The appropriate formulas are as follows

$$Sh = N_i / (\Delta c_{in} DL\pi) \quad (20)$$

$$Nu = Q_c / (\Delta t_{in} \lambda L\pi), \quad (21)$$

where Δc_{in} and Δt_{in} are log-means of the driving forces defined in Eqs (7) and (18).

Theoretical values of the given criteria were computed from the earlier derived relationships⁷

$$Sh = Re_{gr}(f/2)_r^{1/2} F(Sc) / \{20 + F(Sc) + 0.707\phi(Sc) [Re_{gr}(f/2)_r]^{1/4}\} \quad (22)$$

or

$$Nu = Re_{gr}(f/2)_r^{1/2} F(Pr) / \{20 + F(Pr) + 0.707\phi(Pr) [Re_{gr}(f/2)_r]^{1/4}\} \quad (23)$$

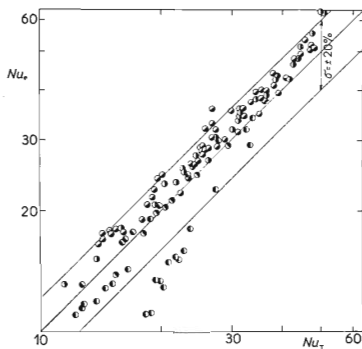


FIG. 3

Comparison of experimental and theoretical value of the Nusselt number: Barnett and Kobe² counter-current flow: water — ○; Cairns and Roper³ counter-current flow: water — ●; Yoshida and Hyodo⁴ counter-current flow: water — ◐; Hikita and coworkers⁵ counter-current flow: water — ◑; Hikita and coworkers⁵ co-current flow: water — ◒

where, according to Malyusov and coworkers¹¹ we have

$$(f/2)_r = (0.89 + 1.74 \cdot 10^{-3} Re_1^{2/3}) 0.041875 / Re_{gr}^{1/4} \quad (24)$$

for the co-current arrangement and for $500 \leq Re \leq 1600$. Further we have

$$(f/2)_r = (1 + 1.74 \cdot 10^{-3} Re_1^{2/3}) 0.041875 / Re_{gr}^{1/4} \quad (25)$$

for the counter-current arrangement and for $30 \leq Re \leq 1600$.

All physical quantities appearing in the computational formulas were taken at the temperature of the film

$$T_f = (T_g + T_l)/2. \quad (26)$$

The results are summarily shown in Table I and Figs 1 to 3.

DISCUSSION

First, one has to realize that the agreement of the theory with experimental results may be anticipated in those cases when the experimental took place under the conditions of adiabatic evaporation, *i.e.* the temperature of liquid did not change on travelling through the column and it could thus be assumed that the temperature of the interface is that of liquid. This conditions are met by the experiments of Hikita and coworkers⁵ which show a good agreement with the theory for the transport of mass and heat as well. This may be apparent from Figs 1 to 3. A somewhat greater variation exhibit data of Yoshida and Hyodo where it could not be ascertained under what conditions the experiments took place since the authors give only the average mean temperatures of liquid. Virtually at constant temperature of liquid were taken the measurements of Cairns and Roper³. Yet, part of their data display significant deviations from the theory. The deviations are the more marked the greater the temperature of liquid (80 to 94°C) and gas (305 to 403°C), which seems to suggest that a local overheating of liquid film could have occurred causing its breakage and decrease of the interfacial area. Greater changes of temperature of liquid are reported for the data of Gilliland and Sherwood¹ and Barnett and Kobe². Nevertheless, the experimental results of the former authors, excepting the data for the co-current flow, agree well with the theory. The data of Barnett differ by 11% both for the heat and mass transfer. Spare for the systematic error, this deviation may be due to the effect of unequal inlet and outlet end temperature of liquid which may distort the driving force. The distortion due to the temperature gradient in the liquid film occurs if the liquid gives off or absorbs heat during evaporation. The extent of this distortion may be different under various conditions and may be affected primarily by the ratio of the heat or mass transfer coefficients and

the conductivity of liquid. No satisfactory explanation could be found for the disagreement of the result of Gilliland and Sherwood for the co-current arrangement of the flow, especially when the data of Hikita and coworkers⁵ for the case of co-current flow were in good agreement with the theoretical results. For a full fledged analysis we missed the scheme of the experimental set-up and construction design of some of the details such as *e.g.* the way of introducing liquid onto the wall of the tube. This might cause turbulization of the gas and an increase of the mass transfer coefficient at the co-current flow.

From Fig. 3 it might be concluded that the data obtained for heat transfer display systematically greater experimental values than the theoretical ones. From Table I, however, it is apparent that the data of Hikita and coworkers⁵, which were the only data obtained under the conditions corresponding to the theoretical model, *i.e.* under the conditions of adiabatic evaporation, well agree with the theory, while for the other data, which constitute the majority one can expect deviations.

It has been found out that the driving force heat transfer, defined by Eq. (18), corresponds with sufficient accuracy to the currently used log-mean driving force. The one defined here is consistently somewhat lower.

TABLE I
Summary of Processed Experimental Data

Reference	Flow arrangement	Evaporated liquid	Range Re_l	Range Re_g	Column $d \cdot 10^3$	$H \cdot 10^3$	$\frac{Sh_e}{Sh_T}$	$\frac{Nu_e}{Nu_T}$
2	A	water	281— 776	2 839—19 174	26.6	1 219	1.11	1.11
3	A	water	252—3 476	2 882— 9 778	21.9	946	0.92	0.81
5	A	water	200— 600	3 890— 8 620	20.0	800	0.98	0.98
4	A	water	309— 750	3 512—14 229	29.0	1 000	0.98	1.09
1	A	water	760—1 179	3 020—11 876	26.7	1 170	1.00	—
1	A	toluene	1 348—1 527	3 771—11 574	26.7	1 170	1.00	—
1	A	ethyl-acetate	1 431—1 458	3 795—12 809	26.7	1 170	0.99	—
1	A	isopropanol	402— 403	4 991— 6 513	26.7	1 170	1.07	—
5	B	water	200— 600	3 560—11 650	2.00	800	1.06	1.05
1	B	water	752—1 194	3 041—26 603	26.7	1 170	1.39	—
1	B	toluene	1 348—1 355	3 466—12 045	26.7	1 170	1.41	—
1	B	ethylacetate	1 345—1 571	3 990—16 134	26.7	1 170	1.34	—
1	B	isopropanol	396— 417	3 035—15 026	26.7	1 170	1.44	—

A — counter- and B — co-current flow

It can be concluded that the values of the Sherwood and the Nusselt number computed from experimental data of various authors and obtained under various conditions display a good agreement with those obtained theoretically on the basis of an earlier published relationship⁷. The explanation of the more significant deviations, which are also in disagreement with the data of other authors, would call for additional information which is not at our disposal.

LIST OF SYMBOLS

<i>A</i>	interfacial area (m^2)
<i>c</i>	concentration of vapours in air (mol m^{-3})
<i>c_p</i>	specific heat at constant pressure ($\text{J kg}^{-1} \text{K}^{-1}$)
<i>D</i>	diffusivity ($\text{m}^2 \text{s}^{-1}$)
<i>d</i>	internal column diameter (m)
<i>f</i>	coefficient of resistance
<i>F</i> (Sc), <i>F</i> (Pr)	function defined in ref. ⁷
<i>G</i>	mass flow rate (kg s^{-1})
<i>g</i>	acceleration due to gravity (m s^{-2})
<i>h</i>	thickness of liquid film (m)
<i>k</i>	mass transfer coefficient (m s^{-1})
<i>L</i>	effective column length (m)
<i>N</i>	rate of evaporation (kg s^{-1}), (mol s^{-1})
<i>P</i>	pressure in column (Pa)
<i>p</i>	partial pressure of component (Pa)
<i>Q</i>	rate of heat transfer (W)
<i>R</i>	gas constant ($\text{J mol}^{-1} \text{K}^{-1}$)
<i>r</i>	heat of evaporation (J kg^{-1})
<i>T</i>	temperature (K)
<i>t</i>	temperature ($^{\circ}\text{C}$)
<i>v</i>	velocity (m s^{-1})
α	heat transfer coefficient ($\text{W m}^{-2} \text{K}^{-1}$)
Γ	density of irrigation ($\text{kg s}^{-1} \text{m}^{-1}$)
Δ	difference of two values
σ	standard deviation
ϕ (Sc), ϕ (Pr)	functions defined in ref. ⁷
λ	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
μ	dynamic viscosity (Pa s)
ν	kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
ρ	density (kg m^{-3})

$$Re_g = v_g d / \nu_g$$

$$Re_{gr} = v_{gr} (d - 2h) / \nu_g$$

$$Re_1 = 4\Gamma / v_1$$

$$Nu = \alpha (d - 2h) / \lambda$$

$$Sh = k (d - 2h) / D$$

$$Sc = (\nu / D)$$

$$Pr = (c_p \mu / \lambda)$$

Subscripts

- a air
- b in turbulent core
- c convection
- f gas film
- g gas phase
- i interface
- l liquid phase
- ln log-mean
- R radiation
- r relative
- t total
- v vapour
- 1 gas inlet
- 2 gas outlet
- * equilibrium value

REFERENCES

1. Gilliland E. R., Sherwood T. K.: *Ind. Eng. Chem.* 26, 516 (1934).
2. Barnett W. I., Kobe K. A.: *Ind. Eng. Chem.* 33, 436 (1941).
3. Cairns R. C., Roper G. H.: *Chem. Eng. Sci.* 3, 97 (1954).
4. Yoshida T., Hyodo T.: *Ind. Eng. Chem. Process Des. Develop.* 9, 207 (1970).
5. Hikita H., Ishimi K., Asano K., Ohba Y.: *Can. J. Chem. Eng.* 57, 578 (1979).
6. Frank-Kamenetskii D. A.: *Diffusion and Heat Transfer in Chemical Kinetics*, 2nd Edition. Plenum Press, New York—London 1969.
7. Kolář V.: *This Journal* 42, 1310 (1977).
8. Nusselt W.: *Z. Ver. Deut. Ing.* 60, 541 (1916).
9. Myasnikov S. K., Kulov N. N., Malyusov V. A., Zhavoronkov N. M.: *Teor. Osn. Khim. Tekhnol.* 6, 893 (1972).
10. Brauer H.: *VDI Forschungsheft* 457 (1956).
11. Malyusov V. A., Myasnikov S. K., Kulov N. N.: *Teor. Osn. Khim. Tekhnol.* 7, 524 (1973).

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